# Getting Ready to Teach Unit 2

## Learning Path in the Common Core Standards

In this unit, students extend their understanding of the base-ten system to decimals to the thousandths place, building on their Grade 4 work with tenths and hundredths.

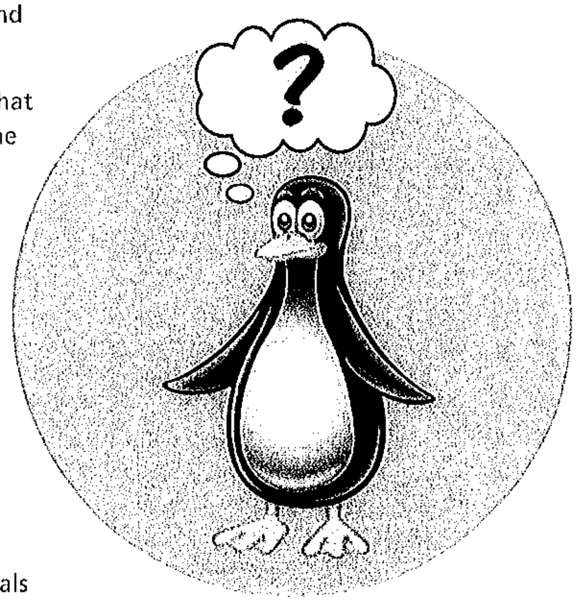
Concrete materials, number lines, and visual drawings are used throughout the unit to illustrate important number and operation concepts. Students observe that the process of composing and decomposing a base-ten unit is the same for decimals as for whole numbers and the same methods of recording numerical work can be used with decimals as with whole numbers.

Visual models and real world situations are used throughout the unit to illustrate important decimal concepts.

## Help Students Avoid Common Errors

*Math Expressions* gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to the students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin.




- **Lesson 3:** Incorrectly writing zeros to make equivalent decimals.
- **Lesson 5:** Not aligning addends by place value when finding a sum
- **Lesson 6:** Subtracting greater digits from lesser digits without ungrouping
- **Lesson 9:** Graphing data using inconsistent axes intervals

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

***Math Expressions***
**VOCABULARY**

As you teach this unit, emphasize understanding of these terms:

- ungrouping
- break-apart drawing

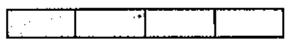
*See the Teacher Glossary*

## Decimals as Equal Divisions

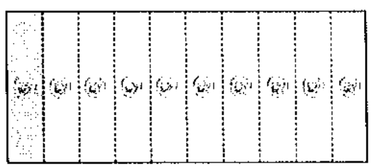
Lesson **1**

**Relating Fractions and Decimals** Visual models are used to show fraction and decimal notations, which are different ways to represent a whole divided into equal parts.

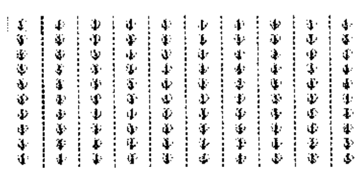
In fraction notation, the denominator represents the number of equal parts a whole is divided into, and the numerator represents the number of parts being described.

$\frac{1}{4}$  1 of 4 equal parts

Decimal notation shows the number of places to the right of the ones place. The tenths place represents the division of 1 whole into 10 equal parts.

$\frac{1}{10}$  1 of 10 equal parts

The hundredths place represents the division of 1 whole into 100 equal parts, and shows each tenth divided into 10 equal parts.

$\frac{1}{100}$  1 of 100 equal parts

The thousandths place represents the division of 1 whole into 1,000 equal parts, and shows each hundredth divided into 10 equal parts.


$\frac{1}{1,000}$ 1 of 1,000 equal parts

$0.001 one-tenth of a penny or one-thousandth of a dollar

Students read decimals as if they are fractions, and use repeated reasoning to recognize patterns such as when a denominator is a power of 10, the number of zeros in the denominator is the number of places to the right of the ones place in an equivalent decimal.

*from* **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

**Understand the Place Value System** In Grade 5, students extend their understanding of the base-ten system to decimals to the thousandths place, building on their Grade 4 work with tenths and hundredths.

## Thousands to Thousandths

Lesson **2**

**Understanding Place Value** A place value chart enables students to see how adjacent place values are related. Place value relationships are used when comparing two or more numbers.

**× 10 (Larger)** **Place Value Chart** **÷ 10 (Smaller)**

| Thousands | Hundreds | Tens | Ones | Tenths | Hundredths | Thousandths |
|---|---|---|---|---|---|---|
| 1,000 | 100 | 10. | 1 | 0.1 | 0.01 | 0.001 |
| $\frac{1,000}{1}$ | $\frac{100}{1}$ | $\frac{10}{1}$ | $\frac{1}{1}$ | $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1,000}$ |
| \$1,000.00 | \$100.00 | \$10.00 | \$1 | \$0.10 | \$0.01 | \$0.001 |

A place value chart also helps students understand ten-for-one ungroupings and one-for-ten groupings when finding sums and differences of whole numbers and decimals. The bottom row of the table shows coins and bills that represent base-ten place value.

**Secret Code Cards** The use of Secret Code Cards provides a way for students to see the expanded form of numbers.

**Modeling Whole Numbers** To model a whole number such as 2,435 students select cards representing 2 thousands, 4 hundreds, 3 tens, and 5 ones. When spread out and arranged in descending order from left to right, the cards show the expanded form of the number.

| 2,000 | 400 | 30 | 5 |
|---|---|---|---|
| **2,000** | **400** | **30** | **5** |
| 2,000 + | 400 + | 30 + | 5 |

Overlapping the cards reveals the standard form of the number.

| 2,000 | 400 | 30 | 5 |
|---|---|---|---|
| **2,** | **4** | **3** | **5** |

The numbers in the upper left corners of the cards represent the number of thousands, hundreds, tens, and ones that make up the whole number.

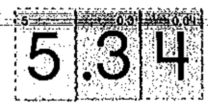
*from* **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

**Generalize place value understanding for multi-digit whole numbers** In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. Because of this, multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left.

**Modeling Decimal Numbers** To model a decimal such as 5.34 students select cards representing 5 ones, 3 tenths, and 4 hundredths. When spread apart and arranged in descending order from left to right, the cards show the expanded form of the number.


**5 + 0.3 + 0.04**

Overlapping the cards reveals the standard form of the number.




The numbers in the upper corners of the cards represent the number of ones, tenths, and hundredths that make up the decimal.

Secret Code Cards help students read whole numbers and decimals and write them in expanded, standard, and word form.

Expanded Form: 5 + 0.3 + 0.04

Standard Form: 5.34

Word Form: five and thirty-four hundredths

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Children can use layered place value cards to see the 10 "hiding" inside any teen number. Such decompositions can be connected to numbers represented with objects and math drawings.

## Equate and Compare Thousandths

Lesson 3

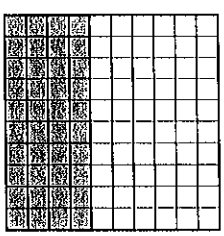
**Comparing Decimals** The tenths and hundredths bars and grids in Lesson 3 represent different ways to model decimals, and provide a way for students to see how two decimal quantities compare.

**Tenths and Hundredths Bars** A bar is used to represent 1 whole, and incorporates tenths and hundredths divisions to represent a part of that whole. The shading below, for example, represents 0.62, and enables students to see the expanded form of 0.62 as 0.6 + 0.02.

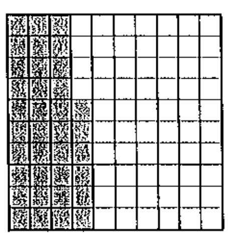
Shading also provides a visual way for students to compare two decimal values. By shading a bar to represent each value, students can see that the bar with a greater number of tenths or hundredths shaded represents the greater value. Comparing tenths and hundredths in this visual way forms the foundation of comparing in a more symbolic way when students compare the digits in the tenths place (i.e., the number of tenths), followed if necessary by comparing the digits in the hundredths place (i.e. the number of hundredths).

**Area Grids** Lesson 3 includes other visual models to enhance understanding of place value. Ten-by-ten grids are used to represent 1 whole. Since the area of each grid is 10 × 10 or 100 unit squares, students shade any number of rows or columns to represent tenths, and any number of unit squares to represent hundredths. To compare 0.4 and 0.36 for example, students shade grids as shown below.

0.4



0.36

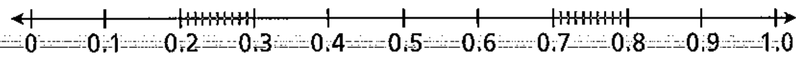


The shading enables students to form conclusions such as these:

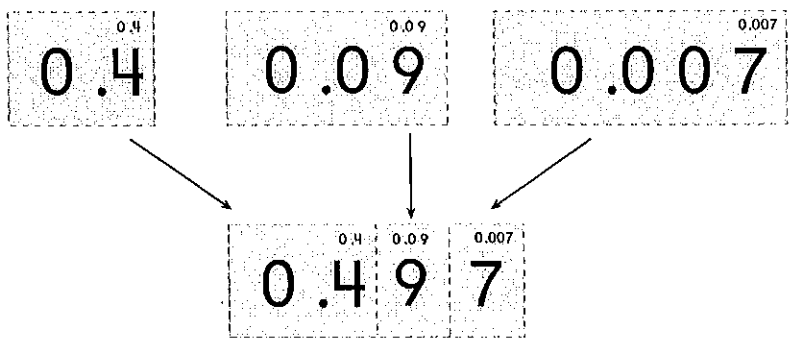
- Since more of the 0.4 grid is shaded, 0.4 > 0.36.
- Since more tenths of the 0.4 grid are shaded, 0.4 > 0.36.
- Since more hundredths of the 0.4 grid are shaded, 0.4 > 0.36.

**Other Ways to Compare** Students also use number lines, Secret Code Cards, and money to enhance their understanding of place value and how decimals compare.

A number line is a model of distance from zero, and is a visual way for students to compare two decimals. Students conclude from their work with number lines that any value on a number line is greater than any value to its left because it consists of more tenths, more hundredths, or more tenths and hundredths.




Secret Code Cards for a decimal such as 0.497 help students see that 0.4 > 0.09 and 0.09 > 0.007.




Students also use coins that represent our base-ten number system (i.e., dimes and pennies) to compare decimals. Since 10 dimes = 1 dollar and 100 pennies = 1 dollar, dimes represent tenths of a dollar and pennies represent hundredths. Modeling with coins leads students to generalize that a greater amount has a greater number of dimes, and when the number of dimes is the same, the greater amount has the greater number of pennies.

Students use a variety of models in Lesson 3 to build and enhance their understanding of place value. Different models appeal to different learning styles and give all students an opportunity to understand the concepts.

## Add and Subtract Whole Numbers and Decimals

Lessons 4 5 6

**Decimal and Whole Number Algorithms** Lessons 4–6 involve using addition and subtraction to find decimal sums and differences. The decimals used for these computations are both greater than and less than 1.

**Place Value** In Lesson 4, the place value chart makes its second appearance. The chart again shows the relationships that adjacent places in our base-ten numeration system share. It also incorporates metric units because the metric system of measure is also a base-ten system. For any two adjacent places or units in either system, the value of the place to the left is 10 times the value of the place to its right, and the value of a place to the right is $\frac{1}{10}$ the value of the place to its left. Both systems involve powers of 10, and the metric conversions that students are asked to complete in this lesson reinforce their understanding of place value.

**Whole Number Addition Strategies** Prior to finding a variety of decimal sums and differences in Lesson 5, students share strategies for finding the sum of two whole numbers. They conclude from the discussion that strategies used to find whole number sums can also be used to find decimal sums. Students then discuss and demonstrate those strategies using 769 and 584 as addends.

| | |
|---|---|
| The *New Groups Below* strategy involves recording groupings under the addends. | $\begin{array}{r} 769 \\ +\ 584 \\ \hline {}^{1\,1\,1} \\ 1{,}353 \end{array}$ |
| The *New Groups Above* strategy involves recording groupings above the addends. | $\begin{array}{r} {}^{1\,1\,1} \\ 769 \\ +\ 584 \\ \hline 1{,}353 \end{array}$ |

The *Subtotal* strategy involves adding hundreds, tens, and ones separately.

| Left to Right: | Right to Left: |
|---|---|
| $\begin{array}{r} 1200 \\ 140 \\ +\ 13 \\ \hline 1{,}353 \end{array}$ | $\begin{array}{r} 13 \\ 140 \\ +\ 1200 \\ \hline 1{,}353 \end{array}$ |

*from* **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

**Add and Subtract Decimals**
Because of the uniformity of the structure of the base-ten system, students use the same place value understanding for adding and subtracting decimals as they used for adding and subtracting whole numbers.

**Decimal Addition Strategies** Similar strategies can be used to find the sum of decimal addends.

| | |
|---|---|
| The *New Groups Below* strategy involves recording groupings under the addends. | $\begin{array}{r} 44.25 \\ +\ \ 8.96 \\ \hline \overset{1}{5}\overset{1}{3}.\overset{1}{2}1 \end{array}$ |
| The *New Groups Above* strategy involves recording groupings above the addends. | $\begin{array}{r} \overset{1}{4}\overset{1}{4}.\overset{1}{2}5 \\ +\ \ 8.96 \\ \hline 53.21 \end{array}$ |

**Connect Decimals to Whole Numbers and Fractions** To successfully find the sum of decimal addends, students must understand the importance of adding like units. A simple way for students to make sure they add like units is to make sure the decimal points in the addends are aligned vertically. Aligning decimal points to add decimals is comparable to finding common denominators when adding fractions.

$$\begin{array}{r} 0.1 \\ +\ 0.01 \\ \hline 0.11 \end{array} \quad \rightarrow \quad \begin{array}{c} \frac{1}{10} = \frac{10}{100} \\ \frac{10}{100} + \frac{1}{100} = \frac{11}{100} \end{array}$$

Students' success with decimal sums also depends on their understanding of ungrouping. Students have already spent a great deal of time ungrouping 10 hundreds as 1 thousand, 10 tens as 1 hundred, and 10 ones as 1 ten. When finding decimal sums, they learn that 10 tenths are grouped as 1 one and 10 hundredths are grouped as 1 tenth.

$$\begin{array}{r} \overset{1}{0}.10 \\ +\ 0.91 \\ \hline 1.01 \end{array} \qquad \frac{10}{100} + \frac{91}{100} = \frac{101}{100} = 1\frac{1}{100}$$

When the number of decimal places in two or more addends is different, some students may find it helpful to write equivalent addends. For example, to add a whole number and a decimal, a decimal point and any number of zeros can be written to the right of the whole number.

$$\begin{array}{r} 0.1 \\ +\ 0.91 \\ \hline 1.01 \end{array} \qquad \begin{array}{c} \frac{1}{10} = \frac{10}{100} \\ \frac{10}{100} + \frac{91}{100} = \frac{101}{100} = 1\frac{1}{100} \end{array}$$

$$1 = 1.0 = 1.00 = 1.000$$

To add two decimals that have a different number of decimal places, any number of zeros can be written to the right of the decimal.

$$0.1 = 0.10 = 0.100$$

Writing equivalent addends not only makes it easier for some students to ensure they add only like units, it also may reduce the likelihood of computation errors.

**Connect Decimals to Whole Number and Fractions** In Lesson 6, students learn that our base-ten numeration system has a uniform structure which enables students to use the same place value understandings to subtract whole numbers and decimals as they used to add whole numbers and decimals. In other words, many of the same strategies can be used to add and subtract. For example, the New Groups Above strategy described earlier can be used.

$$\begin{array}{r} \overset{0}{\cancel{1}}\overset{12}{\cancel{3}}.\overset{18}{\cancel{8}}6 \\ -\ 4.91 \\ \hline 8.95 \end{array}$$

As with decimal addition, students must understand the importance of subtracting like units when subtracting decimals (e.g., ones from ones, tenths from tenths), and this is accomplished by making sure the decimal points are aligned before performing the computation.

Students' success finding decimal differences also depends on their understanding of ungrouping. Students have already spent a great deal of time ungrouping 1 thousand as 10 hundreds, 1 hundred as 10 tens, and 1 ten as 10 ones. When finding decimal differences, they must understand that 1 tenth is ungrouped as 10 hundredths, and 1 hundredth is ungrouped as 10 thousandths.

$0.1 = 0.10$ and $0.01 = 0.010$

## Properties and Strategies

Lesson 7

Students learn that addition properties have many applications.

| Properties | |
|---|---|
| Commutative Property of Addition | $a + b = b + a$ |
| Associative Property of Addition | $(a + b) + c = a + (b + c)$ |
| Distributive Property | $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ |

Mathematics often involves computations, and addition properties can be used to make computations easier to perform. For example, simplifying the expression below from left to right requires grouping 14 tens as 1 hundred and 4 tens.

$$91 + 50 + 50 = 141 + 50 = 191$$

The Associative Property of Addition enables students to add 50 + 50 first, and provides them with an opportunity to perform the entire computation using only mental math.

$$91 + 50 + 50 = 91 + 100 = 191$$

The Distributive Property can be used to simplify many computations that involve multiplication and addition.

$$(800 \cdot 9) + (200 \cdot 9) = (800 + 200) \cdot 9 = 1{,}000 \cdot 9 = 9{,}000$$

**Generalize** Properties of addition can also be used by students in a more general way. In the computation below, for example, students who group 20,000 with 80,000 and 30,000 with 70,000 can perform the entire computation (100,000 + 100,000 + 49,000 = 249,000) using only mental math.

$$\begin{array}{r} 30{,}000 \\ 20{,}000 \\ 80{,}000 \\ 49{,}000 \\ +\ 70{,}000 \\ \hline 249{,}000 \end{array}$$

**Real World Applications** Students also have opportunities in Lesson 7 to use these properties in real world situations.

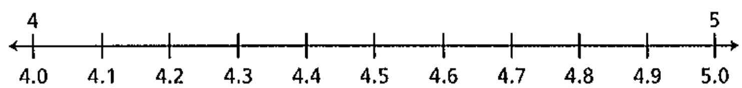
## Round and Estimate With Decimals

Lesson 8

**Rounding** When finding sums and differences, students are not always asked to give exact answers. There are situations in which an estimate (or approximation) of an answer is sufficient.

In Lesson 8, students practice rounding whole numbers and decimals to the nearest one, tenth, and hundredth, as well as infer the place to which a variety of numbers have been rounded.

A number line is one method students can use to round numbers. For example, to round 4.7 to the nearest whole number, students must make decisions such as:




- 4.7 is between the whole numbers 4 and 5.
- 4.5 is halfway between 4 and 5.
- 4.7 is to the right of 4.5.
- 4.7 rounds to 5 because it is closer to 5 than to 4.

Using a number line to round helps students make sense of symbolic rounding, which involves comparing the digit in the place to the right of the rounding place to 5—if the digit is less than 5, the digit in the rounding place does not change, and if the digit is 5 or more, the digit in the rounding place increases by 1.

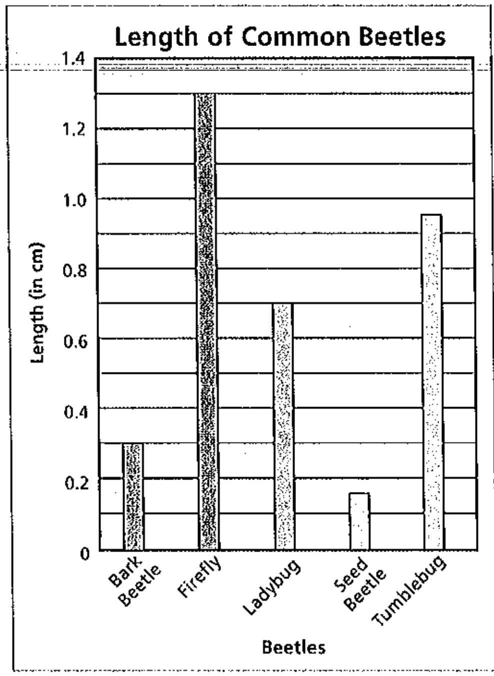
**Estimating** Students should understand the importance of checking computations for reasonableness. One way to check is to apply a property (whenever possible) and perform the computation a second time. However, such a method can involve a great deal of time and be discouraging. A more attractive alternative for students is to estimate an answer. In other words, use their understanding of place value to get a sense of what to expect for an answer. For example, if two decimals are greater than 1 and less than 2, students should expect the sum to be greater than 1 + 1 or 2 and less than 2 + 2 or 4.

Regardless of the method students use to decide reasonableness, all of the methods involve number sense. The more students practice developing number sense, the stronger their understanding of our number system becomes.

## Graph With Decimal Numbers

Lesson 9

In Lesson 9 students examine a decimal bar graph and compare and contrast it to the characteristics of a whole-number bar graph. Students then use the discussion as a springboard to make a bar graph that displays a given set of decimal data.




## Focus on Mathematical Practices

Lesson 10

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about finding decimal sums and differences to compute distances in our solar system.