Methods of Multidigit Division

Lessons 1 2 3

Division Methods In the first several lessons, students explore dividing multidigit whole numbers by one- and two-digit numbers. Students discuss and compare three division methods: Place Value Sections, Expanded Notation, and Digit-by-Digit.

An airplane travels the same distance every day. It travels 3,822 miles in a week. How far does the airplane travel each day?

Place Value Sections

Step 1: 500 — 7 × [3,822 − 3,500] → 322 (Build a new section with each leftover amount.)

Step 2: 500 + 40 — 7 × [3,822 − 3,500 | 322 − 280] → 322, 42

Step 3: 500 + 40 + 6 = 546 — 7 × [3,822 − 3,500 | 322 − 280 | 42 − 42] → 322, 42

Expanded Notation

Step 1: 7)3,822 → 500; −3,500; 322 (Show the zeros in the multipliers.)

Step 2: 7)3,822 → 500, 40; −3,500; 322; −280; 42

Step 3: 7)3,822 → 500, 40, 6 (546); −3,500; 322; −280; 42; −42

Digit-by-Digit

Step 1: 7)3,822 → 5; −3,5; 32 (Put in only one digit at a time.)

Step 2: 7)3,822 → 54; −3,5; 32; −28; 42

Step 3: 7)3,822 → 546; −3,5; 32; −28; 42; −42

Students see that because dividing by a two-digit number requires using estimation, it is easy to overestimate or underestimate a digit of a quotient. They discuss how to adjust estimates when this occurs.

Reasonable Answers Students learn ways to use estimation to determine whether an answer to a division problem is reasonable.

"I know that 1,200 ÷ 6 is 200 and 1,800 ÷ 6 is 300. Because 1,350 is between 1,200 and 1,800, my answer should be between 200 and 300. It is."

6)1,350 = 225

from **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Dividing Multidigit Numbers
Students continue their fourth grade work on division, extending it to computation of whole number quotients with dividends of up to four digits and two-digit divisors. Estimation becomes relevant when extending to two-digit divisors. Even if students round appropriately, the resulting estimate may need to be adjusted.

Interpreting Remainders

Lessons 4 5

Students explore the different ways a remainder can be interpreted.

▶ *The remainder may be dropped or ignored.*

> A roll of ribbon is 1,780 inches long. It takes 1 yard of ribbon (36 inches) to wrap a gift. How many gifts can be wrapped?

1,780 ÷ 36 is 49 R16. The answer is 49 gifts. The remainder is dropped because 16 inches is not enough to wrap a gift.

▶ *The answer may be rounded up to the next whole number.*

> There are 247 people traveling to the basketball tournament by bus. Each bus holds 52 people. How many buses will be needed?

247 ÷ 52 is 4 R39. This represents 4 full buses and 39 extra people. Another bus will be needed for 39 people, so the answer is 5.

▶ *The remainder may be used to form a fraction.*

> The 28 students in Mrs. Colby's class will share 98 slices of pizza equally. How many slices will each student get?

98 ÷ 28 is 3 R14. This means each student gets 3 slices and there are 14 slices left. These leftover slices can be divided, giving each student an additional $\frac{14}{28}$ slice, or $\frac{1}{2}$ slice. The answer is, therefore, $3\frac{1}{2}$ slices.

▶ *The answer may be given as a decimal number.*

> Suppose 16 friends earned $348 at a car wash. They want to divide the money equally. How much should each friend get?

348 ÷ 16 is 21 R12. Each friend gets $21 and there are $12 left. Dividing the $12 gives each friend an additional $\frac{12}{16}$ dollar, which is $\frac{3}{4}$ dollar, or $0.75. The answer is $21.75.

▶ *The remainder may be the answer.*

> A bagel shop has 138 bagels to be packed into boxes of 12 to be sold. The extra bagels are for the workers. How many bagels will the workers get?

138 ÷ 12 is 11 R6. The bagels are packed into 11 boxes. The workers get the 6 extra bagels, so the answer is 6 bagels.

Dividing a Decimal Number by a Whole Number

Lesson 6

Concrete Models In Lesson 6, students divide decimal numbers by one- and two-digit whole numbers. They begin the lesson by using play money to model a division situation. Then they see how their modeling translates to a numerical solution.

Division Methods Students find that the same division methods they use for dividing a whole number by a whole number work for dividing a decimal by a whole number. They also observe that just as when we divide a whole number by a whole number, any leftover amount from one place is ungrouped and moved to the next place.

Three friends set up a lemonade stand and made $20.25. They will share the money equally. Study the steps below to see how much money each person should get.

When the $20 is split 3 ways, each person gets $6. There is $2 left.	We change the $2 to 20 dimes and add the other 2 dimes. There are 22 dimes.	When we split 22 dimes 3 ways, each person gets 7 dimes. There is 1 dime left.	We change the dime to 10 cents and add the other 5 cents. Now we split 15 cents 3 ways.
6 3)20.25 −18 2	6. 3)20.25 −18 2.2	6.7 3)20.25 −18 2.2 −2.1 .1	6.75 3)20.25 −18 2.2 −2.1 .15 −.15

Dividing by a Decimal Number

Lessons 7 8 9

Shift Patterns Students begin Lesson 7 by using the concept of money to explore how the digits in a whole number shift when that number is divided by 0.1, 0.01, or 0.001. In Lesson 8, students do a similar exploration with dividing decimal numbers. In both cases, they find the following:

- Dividing a number by 0.1 means finding the number of tenths in that number. Because there are 10 tenths in every whole, dividing by 0.1 is the same as multiplying by 10. For example, $45.6 \div 0.1 = 45.6 \times 10 = 456$.
- Dividing a number by 0.01 means finding the number of hundredths in that number. Because there are 100 hundredths in every whole, dividing by 0.01 is the same as multiplying by 100. For example, $45.6 \div 0.01 = 45.6 \times 100 = 4{,}560$.
- Dividing a number by 0.001 means finding the number of thousandths in that number. Because there are 1,000 thousandths in every whole, dividing by 0.001 is the same as multiplying by 1,000. For example, $45.6 \div 0.001 = 45.6 \times 1{,}000 = 45{,}600$.

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Dividing by 0.1 and 0.01 As with decimal multiplication, students can first examine the cases of dividing by 0.1 and 0.01 to see that the quotient becomes 10 times or 100 times as large as the dividend.

Connect to Fractions In Lessons 7 and 8, students learn a strategy for dividing by a decimal. They multiply both the dividend and the divisor by a power of 10 (that is, 10, 100, 1,000, and so on) to change the problem to an equivalent problem with a whole number divisor. They know this method works because it is equivalent to rewriting a fraction as an equivalent fraction.

You can use the strategy below to change a division problem with a decimal divisor to an equivalent problem with a whole number divisor.

Discuss each step used to find 6 ÷ 0.2.

Step 1: Write 6 ÷ 0.2 as a fraction. $6 \div 0.2 = \frac{6}{0.2}$

Step 2: Make an equivalent fraction with a whole number divisor by multiplying $\frac{6}{0.2}$ by 1 in the form of $\frac{10}{10}$. Now you can divide 60 by 2. $\frac{6}{0.2} \times 1 = \frac{6}{0.2} \times \frac{10}{10} = \frac{60}{2}$

21. Why is the answer to 60 ÷ 2 the same as the answer to 6 ÷ 0.2?

The fractions $\frac{60}{2}$ and $\frac{6}{0.2}$ are equivalent, so the division problems 60 ÷ 2 and 6 ÷ 0.2 must also be equivalent.

Students see that they do not need to rewrite the division problem in fraction form to use this method. They can simply move the decimal point in both numbers the same number of places.

You can use the strategy of multiplying both numbers by 10 even when a division problem is given in long division format.

Step 1: Put a decimal point after the whole number. $0.2\overline{)6.}$

Step 2: Multiply both numbers by 10, which shifts the digits one place left. Show this by moving the decimal point one place right. Add zeros if necessary. $0.2\overline{)6.0}$

Step 3: Instead of drawing arrows, you can make little marks called carets (^) to show where you put the "new" decimal points. Now divide 60 by 2. $0.2_{\wedge}\overline{)6.0_{\wedge}}$ (30.)

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Division by a Decimal Students can then proceed to more general cases. For example, to calculate 7 ÷ 0.2, students can reason that 0.2 is 2 tenths and 7 is 70 tenths, so asking how many 2 tenths are in 7 is the same as asking how many 2 tenths are in 70 tenths. In other words, 7 ÷ 0.2 is the same as 70 ÷ 2; multiplying both the 7 and the 0.2 by 10 results in the same quotient.

Compare and Contrast Multiplication and Division

Lesson 10

Choosing the Correct Operation In Lesson 10, students start by solving word problems involving multiplication and division with decimals. They must carefully consider the context of each problem to determine which operation to use.

1. A turtle walks 0.2 mile in 1 hour. How far can it walk in 0.5 hour?
   a. Do you need to multiply or divide to solve? multiply
   b. Will the answer be more or less than 0.2 miles? less
   c. What is the answer? 0.1 mile
2. Gus ran 3.6 miles. He took a sip of water every 0.9 mile. How many sips did he take?
   a. Do you need to multiply or divide to solve? divide
   b. Will the answer be greater or less than 3.6? greater
   c. What is the answer? 4 sips

Making Generalizations Students make generalizations about multiplying and dividing a whole number by another number. Specifically, students discuss the following points:

- Multiplying a whole number *a* by a whole number greater than 1 gives a product greater than *a*.
- Multiplying a whole number *a* by a decimal less than 1 gives a product less than *a*.
- Dividing a whole number *a* by a whole number greater than 1 gives a quotient less than *a*.
- Dividing a whole number *a* by a decimal less than 1 gives a quotient greater than *a*.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Dividing by a Decimal Students can summarize the results of their reasoning as specific numerical patterns then as one general overall pattern such as "when the decimal point in the divisor is moved to make a whole number, the decimal point in the dividend should be moved the same number of places."

Predict the Relative Size of the Answer Students solve mixed multiplication and division word problems in which they must first predict the size of the answer relative to the size of one of the numbers in the problem. This requires them to reason about the operation that is required and about the results of multiplying or dividing by a number greater than or less than 1.

18. Farmer Ortigoza has 124.6 acres of land. Farmer Ruben has 0.8 times as much land as Farmer Ortigoza.

    a. Does Farmer Ruben have more or less than 124.6 acres?

    less

    b. How many acres does Farmer Ruben have? 99.68 acres

19. Mee Young has 48 meters of crepe paper. She will cut it into strips that are each 0.6 meter long.

    a. Will Mee Young get more or fewer than 48 strips?

    more

    b. How many strips will Mee Young get? 80 strips

Focus on Mathematical Practices

Lesson 11

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use decimal operations to compute currency exchange rates.