Getting Ready to Teach Unit 7

Learning Path in the Common Core Standards

In this unit, students use algebraic reasoning and coordinate graphs. Their work involves reading, writing, simplifying, and evaluating algebraic expressions, exploring patterns and relationships, and plotting and locating points in the coordinate plane.

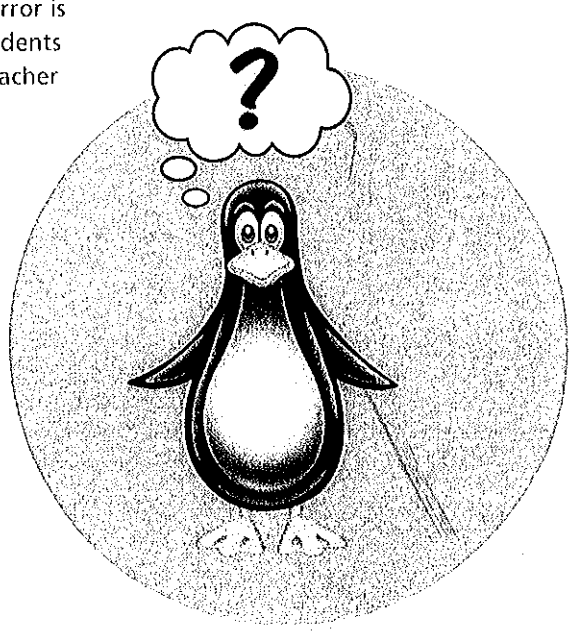
Visual models and real world situations are used throughout the unit to illustrate algebraic thinking.

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin:

- **Lesson 2:** Not writing parentheses to indicate the operation to be performed first
- **Lesson 4:** Incorrectly predicting a term in a pattern
- **Lesson 5:** Transposing *x*- and *y*-coordinates in the coordinate plane



In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Algebraic Reasoning and Expressions

Lessons 1 2 3

Students begin to build their ability to reason algebraically by working with expressions. Expressions use numbers and symbols (such as variables and operation signs) to "express" computations.

Expression	Computation in Words
$3.5 + 6.3$	Add 3.5 and 6.3.
$10 - 2$	Subtract 2 from 10.
$\frac{1}{2} \cdot p$	Multiply p by $\frac{1}{2}$.
$14 \div 4$	Divide 14 by 4.

Students write a computation in words given an expression, and write an expression given a computation in words. Because addition and multiplication are commutative operations, symbolic and word expressions for addition and multiplication can be written in different ways.

Add 14 and t. → $14 + t$ or $t + 14$

Find the product of 12 and 0.1. → 12×0.1 or 0.1×12

Parentheses In Lessons 8–11 in Unit 6, students translated two-step and multistep word problems to equations, and learned the importance of using grouping symbols to indicate the operation to be performed first if an equation contained two or more different operations.

The concept of using parentheses as grouping symbols is emphasized in the second activity in Lesson 1, and in Lessons 2 and 3 of this unit.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Expressions and Equations As preparation for the Expressions and Equations Progression in the middle grades, students in Grade 5 begin working more formally with expressions. They write expressions to express a calculation, e.g., writing $2 \times (8 + 7)$ to express the calculation "add 8 and 7, then multiply by 2." They also evaluate and interpret expressions, e.g., using their conceptual understanding of multiplication to interpret $3 \times (18{,}932 + 921)$ as being three times as large as $18{,}932 + 921$, without having to calculate the indicated sum or product. Thus, students in Grade 5 begin to think about numerical expressions in ways that prefigure their later work with variable expressions (e.g., three times an unknown length is $3 \cdot L$). In Grade 5, this work should be viewed as exploratory rather than for attaining mastery; for example, expressions should not contain nested grouping symbols, and they should be no more complex than the expressions one finds in an application of the associative or distributive property, e.g., $(8 + 27) + 2$ or $(6 \times 30) + (6 \times 7)$. Note however that the numbers in expressions need not always be whole numbers.

Expressions with More than One Operation Lesson 1 also introduces students to expressions that involve more than one operation and the concept of the Order of Operations.

Order of Operations

Step 1 Perform operations inside parentheses.
Step 2 Multiply and divide from left to right.
Step 3 Add and subtract from left to right.

Students use the Order of Operations to indicate, by using words or by writing an expression with parentheses, the operation to be performed first.

10. Consider the expression $12 \div (5 + 2)$.

    a. Which operation is done first, division or addition?
    addition

    b. Write the computation in words.
    Possible answer: Divide 12 by the sum of 5 and 2.

11. Consider the expression $12 \div 5 + 2$.

    a. Which operation is done first, division or addition?
    division

    b. Write the computation in words.
    Possible answer: Divide 12 by 5 and then add 2.

Write the computation in words or the expression for the words. Think about the Order of Operations.

12. $3.5 - (2.1 + 1.2)$ Subtract the sum of 2.1 and 1.2 from 3.5.
13. $\frac{1}{2} + \frac{3}{4} \cdot t$ Add $\frac{1}{2}$ to the product of $\frac{3}{4}$ and t.
14. $(25 - 10) \div 5$ Subtract 10 from 25 and then divide by 5.
15. Multiply the sum of p and 3 by 0.1. $(p + 3) \cdot 0.1$ or $0.1 \cdot (p + 3)$

A great deal of the mathematics students will learn and perform in their future studies will involve, in some way, the Order of Operations.

Simplify Expressions The activities in Lesson 2 involve simplifying expressions, and students learn that simplifying an expression means finding its value.

A common error when simplifying expressions is to ignore the Order of Operations and instead, perform the operations as they appear from left to right. It is important for students to understand many expressions will not simplify correctly if the Order of Operations is ignored. For example, the expression below simplifies to 18 if the operations are performed left to right.

$$12 - 3 \cdot 2$$

To accustom themselves to following the Order of Operations, students first simplify an expression in a step-by-step fashion.

1. Follow the Order of Operations to simplify $25 - (5 + 2) \cdot 3$.

Step 1 Perform operations inside parentheses.	$25 - 7 \cdot 3$
Step 2 Multiply and divide from left to right.	$25 - 21$
Step 3 Add and subtract from left to right.	4

Students then simplify expressions without prompts, gaining experience working within the Order of Operations.

Simplify. Follow the Order of Operations.

2. $5 + 16 \div 4$	3. $10 \cdot (0.3 + 0.2)$	4. $20 \div 4 + 3 \cdot 3$
9	5	14
5. $(\frac{5}{6} - \frac{1}{3}) \cdot 4$	6. $21 - 12 + 9 - 2$	7. $6 \times (2 + 4) \div 3$
2	16	12
8. $0.3 + 0.1 \cdot 5 + 0.2$	9. $18 + 9 \div 0.1$	10. $36 \div 3 \cdot 2$
1	108	24

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Connection to Expressions and Equations In Grade 6, students will begin to view expressions not just as calculation recipes but as entities in their own right, which can be described in terms of their parts. For example, students see $8 \cdot (5 + 2)$ as the product of 8 with the sum $5 + 2$. In particular, students must use the conventions for order of operations to *interpret* expressions, not just to evaluate them. Viewing expressions as entities created from component parts is essential for seeing the structure of expressions in later grades and using structure to reason about expressions and functions.

As noted above, the foundation for these later competencies is laid in Grade 5 when students write expressions to record a "calculation recipe" without actually evaluating the expression, use parentheses to formulate expressions, and examine patterns and relationships numerically and visually on a coordinate plane graph. Before Grade 5, student thinking that also builds toward the Grade 6 [Expressions & Equations] work is focusing on the expressions on each side of an equation, relating each expression to the situation, and discussing the situational and mathematical vocabulary involved to deepen the understandings of expressions and equations.

Evaluate Expressions In Lesson 3, students' algebraic reasoning skills evolve to include expressions with variables. Variables represent unknown numbers. The concept of variables is not new to students. They have worked with formulas that involve variables in the past, such as using $A = l \cdot w$ and $P = 2l + 2w$ to compute the area and perimeter of a rectangle.

In this lesson students learn about evaluating expressions. Evaluating an expression means substituting values for the variables and then performing the operations using the Order of Operations.

Evaluate $5 \cdot (p - 2)$ for $p = 10$.

$5 \cdot (p - 2)$	$= 5 \cdot (10 - 2)$	Substitute 10 for p.
	$= 5 \cdot 8$	Subtract inside parentheses.
	$= 40$	Multiply.

The expressions students evaluate include fractions, decimals and whole numbers.

1. $m - 4.7$ for $m = 10$

   5.3

2. $5 \div x$ for $x = \frac{1}{3}$

   15

3. $5 + n \cdot 4$ for $n = 3$

   17

4. $\frac{1}{5} \cdot x$ for $x = 15$

   3

5. $7.5 \times (d - 2.5)$ for $d = 3.5$

   7.5

6. $48 \div (z - 6)$ for $z = 14$

   6

7. $10 \cdot (0.05 + q)$ for $q = 1.2$

   12.5

8. $2\frac{3}{4} + d - 1\frac{1}{4} + 5\frac{1}{2}$ for $d = 1\frac{1}{2}$

   $8\frac{1}{2}$

9. $1{,}000 \cdot h$ for $h = 0.004$

   4

10. $(t + 18) \div 5$ for $t = 17$

    7

11. $54 \div 3 \cdot v$ for $v = 3$

    54

12. $6 \cdot 0.01 + n \cdot 0.1$ for $n = 2$

    0.26

Writing Expressions A variety of real world contexts provide students with opportunities to apply their understanding of expressions.

13. Four friends earned \$24 by washing cars and m dollars by mowing lawns. They want to divide the total equally.

    a. Write an expression for the amount each friend gets.

    $(24 + m) \div 4$

    b. If they made \$50 mowing lawns, how much should each friend get?

    \$18.50

14. There are $\frac{2}{3}$ as many students in science club as in math club.

    a. If there are m students in math club, how many are in science club?

    $\frac{2}{3} \cdot m$

    b. If there are 27 students in math club, how many are in science club?

    18

15. Kima's cat weighs 6 pounds more than her rabbit. Her dog weighs 3 times as much as her cat. Let r be the weight of Kima's rabbit.

    a. How much does her cat weigh?

    $r + 6$ pounds

    b. How much does her dog weigh?

    $3 \cdot (r + 6)$ pounds

    c. If Kima's rabbit weighs 5 pounds, how much do her cat and dog weigh?

    cat: 11 pounds; dog: 33 pounds

16. To change a temperature from degrees Celsius to degrees Fahrenheit, multiply it by $\frac{9}{5}$ and then add 32.

    a. Let c be a temperature in degrees Celsius. Write an expression for changing c to degrees Fahrenheit.

    $\frac{9}{5} \cdot c + 32$

    b. Use your expression to change 20°C to Fahrenheit degrees.

    68°F

Patterns and Relationships

Lessons 4 5 6

Patterns Work with patterns begins with numerical sequences (or progressions) of numbers. In each sequence, consecutive numbers (i.e., terms) share the same relationship.

The initial sequence 3, 5, 7, 9, 11, . . . is an example of an arithmetic sequence. A sequence is arithmetic if the difference between consecutive terms is constant.

	3	5	7	9	11
Expressions	3	3 + 2	3 + 2 + 2	3 + 2 + 2 + 2	3 + 2 + 2 + 2 + 2
	3	3 + (1 · 2)	3 + (2 · 2)	3 + (3 · 2)	3 + (4 · 2)

Numerical expressions can be written to describe the terms of the sequence above. Writing expressions gives students a way to find the sixth term of this sequence, or the next term of any sequence. The expressions used to predict 13, the next term of the sequence above, are 3 + 2 + 2 + 2 + 2 + 2 or 3 + 5 • 2.

Students begin their work with numerical sequences by being given the rule of the sequence.

2. a. Write the first five terms of a numerical pattern that begins with 5 and then adds 5.

   5, 10, 15, 20, 25

   b. Write an expression for the sixth term of the pattern.

   5 + 5 + 5 + 5 + 5 + 5 or 6 · 5

   c. Write the sixth term. 30

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**

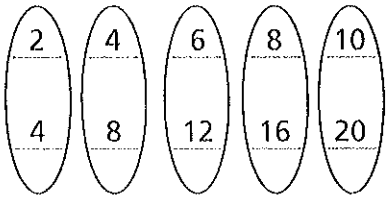
Patterns Students extend their Grade 4 pattern work by working briefly with two numerical patterns that can be related and examining these relationships within sequences of ordered pairs and in the graphs in the first quadrant of the coordinate plane. This work prepares students for studying proportional relationships and functions in middle school.

Patterns and Relationships Students progress from working with one number sequence to working with two. Working with two number sequences gives students an opportunity to think in a proportional way. This work helps form the conceptual foundation for their future studies involving functions and a variety of proportional relationships.

An important part of working with two number sequences is identifying the corresponding terms of the sequences. To identify the corresponding terms, students need to view the terms of the patterns in an ordinal sense, in other words, as *positional*. All terms occupying the same position are corresponding terms, so the terms in the first position are corresponding, the terms in the second position are corresponding, and so on. Drawing circles helps students see the corresponding terms and understand how they are related.

4. a. Write the first five terms of a pattern that begins with 2, and then adds 2.

   b. Write the first five terms of a pattern that begins with 4, and then adds 4.



   c. Circle the corresponding pairs of terms in the patterns. How does each term in the top pattern compare to the corresponding term in the bottom pattern?

   The top term is the bottom term divided by 2.

   d. How does the bottom term compare to the top term?

   The bottom term is the top term times 2.

6. a. Write the first five terms of two different patterns. Sample answers shown.

   1 2 3 4 5

   10 20 30 40 50

   b. Circle the corresponding terms in your patterns. Then describe two different relationships that the corresponding terms of your patterns share.

   The top term is the bottom term divided by 10;

   the bottom term is the top term times 10.

Real World Patterns Students also work with real world examples of number patterns. Presenting the data in tables makes it easier for students to locate the corresponding terms and identify the relationship those terms share.

Overdue Book Late Fee					
Number of Days Late	1	2	3	4	5
Late Fee	15¢	30¢	45¢	60¢	75¢

7. Describe the relationship between the corresponding terms.

Sample answer: The late fee in cents is the number of days late multiplied by fifteen.

Complete the table and describe the relationship between corresponding terms.

8.

Bicycles and Wheels					
Bicycles	1	2	3	4	5
Wheels	2	4	6	8	10

Sample answer: The number of wheels is the number of bicycles multiplied by two.

9.

Cost of Concert Tickets					
Tickets	1	2	3	4	5
Cost in Dollars	35	70	105	140	175

Sample answer: The cost of tickets in dollars is the number of tickets multiplied by thirty-five.

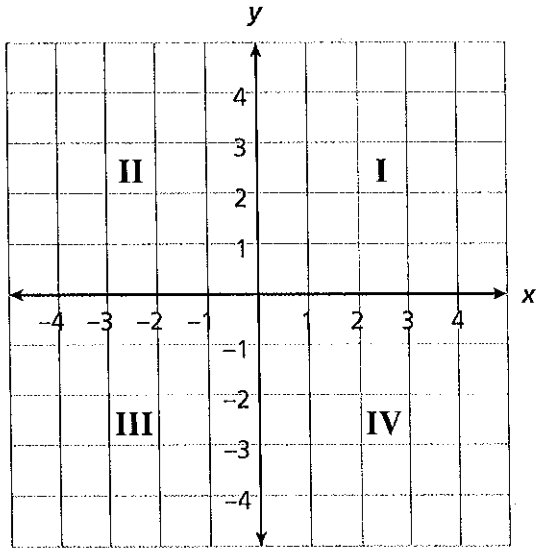
10.

Weather Relationships					
Inches of Rain	0	0.5	1	1.5	2
Inches of Snow	0	5	10	15	20

Sample answer: The number of inches of snow is the number of inches of rain multiplied by ten.

The Coordinate Plane In Lesson 5 students are introduced to the coordinate plane. The plotting and locating of points in the plane is an extension of their work with the proportional relationships of number sequences in Lesson 4.

A coordinate plane is a two-dimensional system that uses numerical coordinates to specify points, or locations, in the plane. It is important for students to understand the coordinates of the form (x, y) as measures of distance, with the x-coordinate representing perpendicular distance from the y- or vertical axis, and the y-coordinate representing perpendicular distance from the x- or horizontal axis. In Grade 5, work in the coordinate plane is limited to Quadrant I.

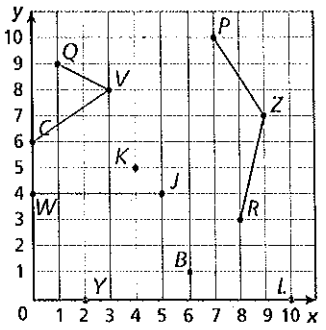


In addition to plotting and locating points in the plane, Problem 31 makes students aware of the common error of transposing the coordinates of an ordered pair as they identify and explain to Puzzled Penguin how to correct the error.

31. Write a response to Puzzled Penguin.

No, point (7, 4) is not correct because it would not form a square. You listed the coordinates in the wrong order. The correct point has an x-coordinate of 4 and a y-coordinate of 7. Its ordered pair would be (4, 7).

The coordinate plane activities also include those that are more open-ended.



Plot and label a point at each location.

20. point *P* at (7, 10)

21. point *C* at (0, 6)

22. point *Z* at (9, 7)

On the coordinate plane above, draw an angle of the given type. The angle should have its vertex at one labeled point and sides that pass through two other labeled points. Give the name of the angle.

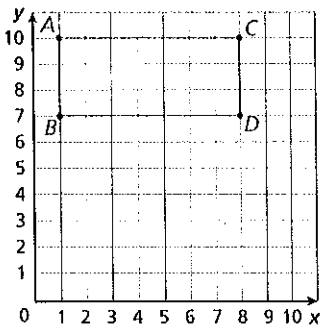
Answers will vary; sample answers shown.

23. acute angle ∠QVC

24. obtuse angle ∠PZR

25. right angle ∠CWJ

Students also compute horizontal and vertical distance.



26. Explain how subtraction can be used to find the lengths of line segments *AB* and *AC*.

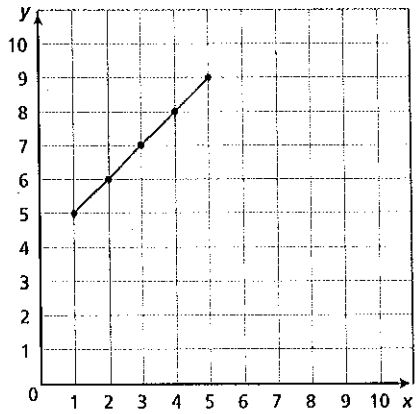
To find the length of segment *AB*, subtract the *y*-coordinates of *B* from *A*; 10 − 7 = 3. To find the length of segment *AC*, subtract the *x*-coordinates of *A* from *C*; 8 − 1 = 7.

Generate and Graph Ordered Pairs The activities in Lesson 6 are again related to the concept of proportional relationships. Although students are not told they are working with functions in this lesson, they are working with a type of function known as a *linear* function. The graph of a linear function is a straight line.

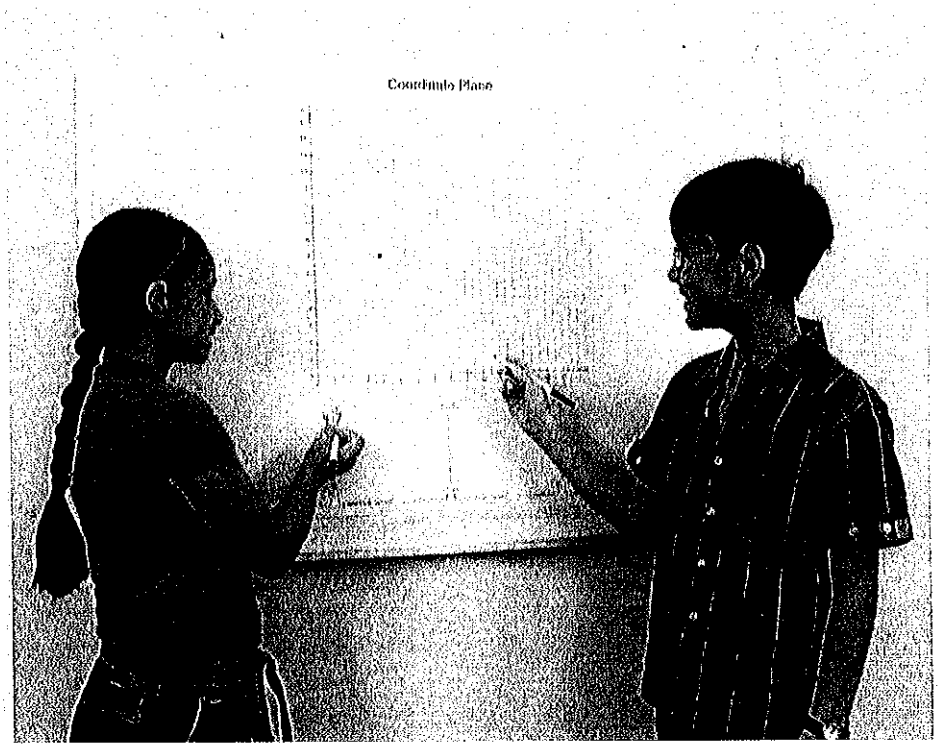
Numerical patterns can be written horizontally or vertically. The *add 4* table below shows a numerical pattern in the left column and the result of adding 4 in the right column.

add 4	
1	5
2	6
3	7
4	8
5	9

(*x*, *y*)
(1, 5)
(2, 6)
(3, 7)
(4, 8)
(5, 9)

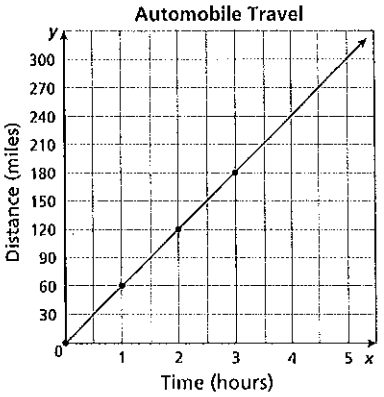


1. Complete the *add 4* table.
2. Complete the (*x*, *y*) table to show the ordered pairs that the *add 4* table represents.
3. Each ordered pair represents a point in the coordinate plane. Graph and connect the points.



Real World Connections A constant change over time is a real world example of a proportional relationship. Students complete their work with proportional relationships by solving real world problems.

The graph represents an automobile traveling at a constant speed.



9. The points on the graph represent four ordered (x, y) pairs. Write the ordered pairs.

(0, 0) (1, 60) (2, 120) (3, 180)

10. Complete the table to show the relationship between time and distance.

Time (hours)	0	1	2	3
Distance (miles)	0	60	120	180

11. At what constant rate of speed was the automobile traveling? Explain how you know.

60 mph; Sample explanation: The graph passes through the point at (1 hour, 60 miles).

Focus on Mathematical Practices

Lesson 7

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use coordinate graphs to explore constellations.