Getting Ready to Teach Unit 8

Learning Path in the Common Core Standards

In Grade 4, students used multiplication to convert larger units to smaller units within the same measurement system. In this unit, students use both multiplication and division to convert within the same system. This is the first grade level in which students convert smaller units to larger units.

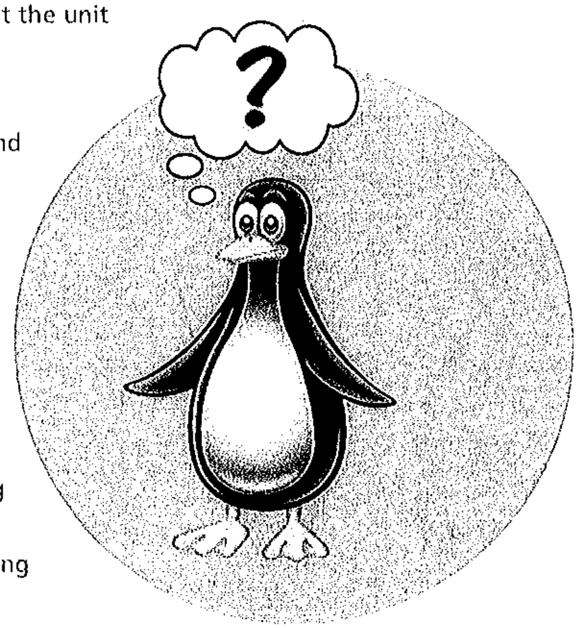
Students worked with the perimeter and area of rectangles in earlier grades. At this grade level, they work with the key concept of volume, exploring the concept using hands-on unit cubes, and progressing in their work to using a formula.

Students also have previous experience identifying geometric figures by their properties. In Grade 5, students draw as well as sort and classify polygons by their attributes. They begin to formulate the idea of a hierarchy of quadrilateral properties.

Visual models and real world situations are used throughout the unit to illustrate important concepts.

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.



In this unit we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin:

- **Lesson 1:** Not doubling length and width when finding perimeter of a rectangle
- **Lesson 9:** Counting only visible unit cubes when counting is used to find volume
- **Lesson 16:** Not recognizing characteristics of polygons

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Converting Measurements

Lessons 1 2 3 4 5 6

Metric Units of Measure Generally speaking, it is simpler to convert one metric unit of measure to another than it is to convert one customary unit of measure to another. Converting customary units requires multiplying or dividing by a wide variety of numbers. Converting metric units requires only multiplying or dividing by a power of 10. Powers of 10 include 10^1 or 10, 10^2 or 100, 10^3 or 1,000, and so on.

Multiplying or dividing by a power of 10 produces a result that is the same as shifting the digits in the measurement a number of places to the left or the right.

Dividing by:

10^1 shifts the digits 1 place to the right.

10^2 shifts the digits 2 places to the right.

10^3 shifts the digits 3 places to the right.

And so on.

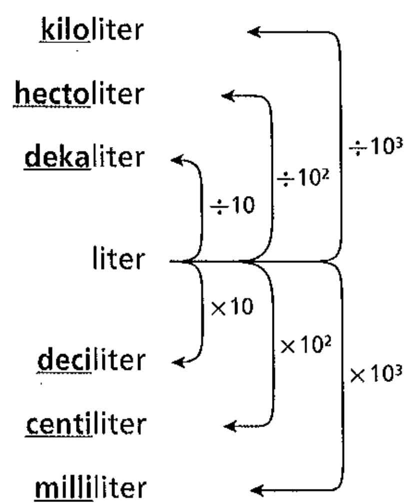
Multiplying by:

10^1 shifts the digits 1 place to the left.

10^2 shifts the digits 2 places to the left.

10^3 shifts the digits 3 places to the left.

And so on.



Generalizations When completing the activities in Lessons 1–3, students should develop an understanding of the following generalizations for converting metric units of measure.

- Multiplication is used to convert a larger unit to a smaller unit.
- Division is used to convert a smaller unit to a larger unit.

Once students become familiar with the relationships that metric units of measure share (i.e., How are meters related to kilometers and millimeters?), they often can perform a variety of metric conversions using only mental math.

Metric Units of Length The metric units of length students work with and convert in this unit include millimeters (mm), centimeters (cm), decimeters (dm), meters (m), dekameters (dam), hectometers (hm), and kilometers (km).

In the metric system, the meter is the basic unit of length. The relationships shown below—comparing 1 meter to other metric units of length and 1 of other metric units of length to meters—are used by students to perform conversions.

Metric Units of Length	
1 dekameter (dam) = 10 meters	1 meter = 0.1 dekameter
1 hectometer (hm) = 100 meters	1 meter = 0.01 hectometer
1 kilometer (km) = 1,000 meters	1 meter = 0.001 kilometer
1 meter = 10 decimeters (dm)	0.1 meter = 1 decimeter
1 meter = 100 centimeters (cm)	0.01 meter = 1 centimeter
1 meter = 1,000 millimeters (mm)	0.001 meter = 1 millimeter

Initially, students use guided examples to convert, and progress to conversions with no guidance.

Example 1 Convert to a Smaller Unit

2 km = ____ m

Step 1: Choose multiplication because we will need more of the smaller units.

Step 2: Multiply by 1,000 because 1,000 m = 1 km.

2 km = 2,000 m (2 × 1,000 = 2,000)

Example 2 Convert to a Larger Unit

50 cm = ____ m

Step 1: Choose division because we will need fewer of the larger units.

Step 2: Divide by 100 because 100 cm = 1 m.

50 cm = 0.5 m (50 ÷ 100 = 0.5)

Complete.

1. 15 m = ____mm
2. 877 cm = ____ m
3. 450 m = ____ km
4. 2.39 m = ____ cm
5. 2,040 mm = ____ m
6. 8.6 km = ____ m

UNIT 8 MATH BACKGROUND

Two-Step and Multistep Problems Exercises and problems in Lesson 1 range from straightforward conversions (such as those shown on the previous page) to multistep problems in real world contexts.

14. Natasha ran 3.1 kilometers. Tonya ran 4 meters more than half as far as Natasha. How many meters did Tonya run? 1,554 meters

24. Mattie is making a collar for her dog. She needs to buy some chain, a clasp, and a name tag. She wants the chain to be 40 centimeters long. A meter of the chain costs $9.75. The clasp is $1.29, and the name tag is $3.43. How much will it cost to make the collar? Estimate to check if your answer is reasonable.

$\$8.62$; Estimate: $(\frac{1}{2} \times \$10) + \$1 + \$3 = \$9$

Liquid Volume and Mass As with length, the concepts of liquid volume and mass in Lessons 2 and 3 are introduced with charts that show the relationships between the basic units, liters and grams, and the other units of liquid volume and mass.

Metric Units of Liquid Volume	
1 dekaliter (daL) = 10 liters	1 liter = 0.1 dekaliter
1 hectoliter (hL) = 100 liters	1 liter = 0.01 hectoliter
1 kiloliter (kL) = 1,000 liters	1 liter = 0.001 kiloliter
1 liter = 10 deciliters (dL)	0.1 liter = 1 deciliter
1 liter = 100 centiliters (cL)	0.01 liter = 1 centiliter
1 liter = 1,000 milliliters (mL)	0.001 liter = 1 milliliter

Metric Units of Mass	
1 dekagram (dag) = 10 grams	1 gram = 0.1 dekagram
1 hectogram (hg) = 100 grams	1 gram = 0.01 hectogram
1 kilogram (kg) = 1,000 grams	1 gram = 0.001 kilogram
1 gram = 10 decigrams (dg)	0.1 gram = 1 decigram
1 gram = 100 centigrams (cg)	0.01 gram = 1 centigram
1 gram = 1,000 milligrams (mg)	0.001 gram = 1 milligram

For these concepts, students again use guided examples to convert, and progress to conversions with no guidance and then to solving multistep problems in real world contexts.

Customary Units of Measure Although the strategies of using multiplication to change to a smaller unit and using division to change to a larger unit are the same for both metric and customary conversions, computations for customary conversions are more complicated because they do not involve powers of 10. In other words, performing customary conversions is not as simple as shifting digits to the left or to the right. For example, changing millimeters (the smallest metric unit of length) to kilometers (the largest unit) simply involves shifting the digits six places to the right (i.e., dividing by 10^6 or 1,000,000). The related customary conversion of inches to miles would typically involve first dividing by 12 to find the number of feet, then dividing by 5,280 to change the number of feet to miles. Metric conversions by comparison are very straightforward.

Length, Liquid Volume, and Weight To successfully convert customary units of length, liquid volume, and weight in Lessons 4–6, students must know a wide range of customary relationships.

Customary Units of Length	
1 foot (ft) = 12 inches (in.)	1 yard (yd) = 3 feet = 36 inches
1 mile (mi) = 1,760 yards = 5,280 feet	

Customary Units of Liquid Volume
1 gallon (gal) = 4 quarts (qt) = 8 pints (pt) = 16 cups (c)
$\frac{1}{4}$ gallon = 1 quart = 2 pints = 4 cups
$\frac{1}{8}$ gallon = $\frac{1}{2}$ quart = 1 pint = 2 cups

Customary Units of Weight	
1 pound (lb) = 16 ounces (oz)	1 ton (T) = 2,000 pounds

UNIT 8 MATH BACKGROUND

As with metric conversions, students initially use guided examples to convert customary units.

Example 1 Convert to a Smaller Unit

15 yd = ____ ft

Step 1: Choose multiplication because we will need more of the smaller units.

Step 2: Multiply by 3 because 3 ft = 1 yd.

15 yd = 45 ft ($15 \times 3 = 45$)

Example 2 Convert to a Larger Unit

104 pt = ____ gal

Step 1: Choose division because we will need fewer of the larger units.

Step 2: Divide by 8 because 8 pt = 1 gal

104 pt = 13 gal ($104 \div 8 = 13$)

Complete.

1. $\frac{1}{2}$ yd = ____ ft
2. ____ T = 14,000 lb
3. ____ oz = $5\frac{1}{4}$ lb
4. ____ yd = 144 in.
5. $2\frac{1}{2}$ qt = ____ pt
6. ____ gal = 48 qt
7. What fraction of 1 quart is 1 cup?
8. What fraction of 1 gallon is 3 pints?

Two-Step and Multistep Problems Word problems related to customary length, liquid volume, and weight in Lessons 4–6 include real world contexts.

11. A $\frac{1}{4}$-lb package of sunflower seeds costs 79¢. An 8-ounce package costs $1.59. Which package represents the lower cost per ounce?

the $\frac{1}{4}$-lb package

16. A serving size for a glass of pineapple-orange punch is $\frac{1}{2}$ cup. Liam needs to make 72 servings of punch. He will use 8 pints of pineapple juice. The rest is orange juice. How many pints of orange juice does he need to make the punch?

10 pints

Fractions Conversions involving customary measurements often include fractional units. For example, the height of a wall is more likely to be stated as $8\frac{1}{2}$ feet instead of 8 feet 6 inches, and a distance jogged is more likely to be stated as $2\frac{1}{4}$ miles instead of 2.25 miles, which in spoken form is *two and twenty-five hundredths miles*. Students working with customary measurements and converting customary units need an understanding of how to find products and quotients when the computations include fractions.

Mass vs. Weight Although mass and weight are different concepts, the terms are often used interchangeably in general, everyday usage. The terms are used properly when *mass* refers to a measurement of the amount of matter an object contains, and *weight* refers to a measurement of the pull of gravity on an object.

A way for students to understand this difference is to present the terms in a real world context. For example, a bowling ball here on Earth has a given mass and weight; suppose that its weight is 12 pounds. If placed on a scale on the moon, where the force of gravity is about $\frac{1}{6}$ that of Earth, the bowling ball would have a weight of about $\frac{1}{6}$ of 12, or 2 pounds. However, in both locations, the mass, or amount of matter that makes up the ball, is unchanged. Stated a different way, weight changes with location; mass does not.

Number Sense Using both metric and customary measures in real world contexts enables students to help make sense of the measures and better understand the relationships that the measures share.

Line Plots

Lesson 7

A line plot is a measure of frequency. Stated a different way, a line plot is a visual display that shows how often something occurs.

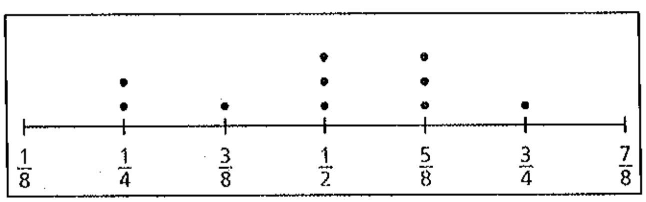
The symbols of a line plot typically include dots or X's although students can use any mark to record data. The symbols used in *Math Expressions* are dots. A key concept for students to understand is that the symbols display a 1-to-1 relationship to frequency. For example, a line plot showing the number of votes each candidate in a class election received would display the same number of symbols as the number of students that voted.

The horizontal axis of a line plot can display whole numbers or fractions, as shown below. Using given sets of data, students complete both types of plots in Lesson 7. The number of symbols above each whole number or fraction is the measure of frequency.

from **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON MEASUREMENT AND DATA**

To make a line plot from the data in the table, the student can ascertain the greatest and least values in the data. [. . .] The student can draw a segment of a number line diagram that includes these extremes, with tick marks indicating specific values on the measurement scale. This is just like part of the scale on a ruler.

6. For 10 days, Mario measured the amount of food that his cat Toby ate each day. The amounts Mario recorded are shown in the table at the right. Graph the results on the line plot.



Cups of Food Toby Ate Each Day

Amounts Toby Ate Each Day for 10 Days	
$\frac{1}{4}$ c	II
$\frac{3}{8}$ c	I
$\frac{1}{2}$ c	III
$\frac{5}{8}$ c	III
$\frac{3}{4}$ c	I

Students' work with line plots also requires them to interpret the data.

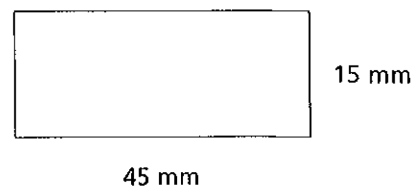
a. What is the total amount of food Toby ate over the 10 days? Explain how you got your answer.

b. What amount of food would Toby get if it were distributed evenly each day over the 10 days?

Perimeter and Area

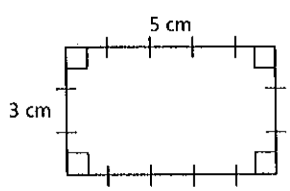
Lesson 8

When students use the length and the width of a rectangle to calculate its perimeter or area, they sometimes don't know which dimension represents the length and which dimension represents the width. Given the rectangle at the right, for example, some students may assume that 45 mm, or the side-to-side measure, is the width of the rectangle, while others may generalize that 15 mm represents the width because the length of a rectangle must always be greater than its width.



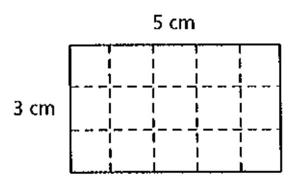
Because addition and multiplication are commutative (i.e., the order of the addends or factors does not change the sum or product), the length and width measures of a rectangle can be used interchangeably when finding perimeter or area. However, there is some consensus in the math community that length is the longer dimension of an object.

Conceptual Understanding Tick marks and unit squares help students build a conceptual understanding of perimeter and area.



$P = 3 \text{ cm} + 5 \text{ cm} + 3 \text{ cm} + 5 \text{ cm} = 16 \text{ cm}$

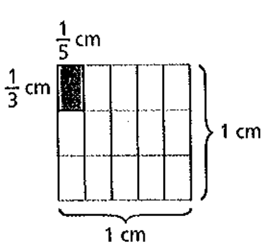
Formula: $P = 2l + 2w$



$A = 3 \text{ cm} \times 5 \text{ cm} = 15 \text{ sq cm}$

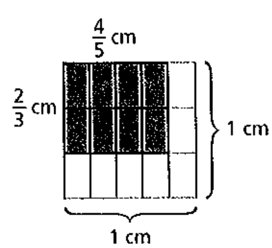
Formula: $A = l \times w$

This conceptual understanding helps when students work with fractional side lengths.



A = 1 of 15 equal parts

$A = \frac{1}{3} \text{ cm} \times \frac{1}{5} \text{ cm} = \frac{1}{15} \text{ sq cm}$



A = eight $\frac{1}{15}$'s

$A = \frac{2}{3} \text{ cm} \times \frac{4}{5} \text{ cm} = \frac{8}{15} \text{ sq cm}$

Volume

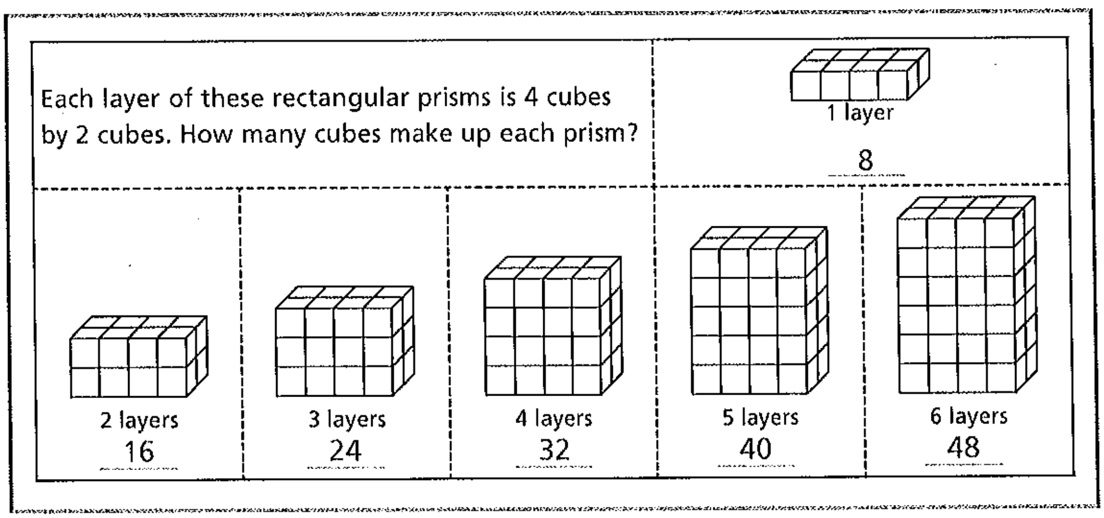
Lessons 9 10 11 12 13

Cubic Units Nets are used to explore the concept of volume and introduce students to the idea of three-dimensional thinking. Students tape and fold each net so that one face remains open. Then they fill the net with unit cubes and count the cubes to find the volume of the prism.

Volume Concepts As students explore volume, they discuss these volume concepts.

- ▶ A cube with side length 1 unit is called a "unit cube."
- ▶ A solid figure that can be packed without gaps or overlaps using *n* unit cubes has a volume of *n* cubic units.
- ▶ The same-sized cubes must be used to determine volume.

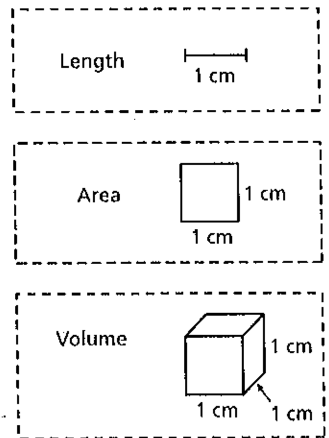
Volume Formula The hands-on activity above, along with a variety of exercises (such as the one shown below), enable students to begin to think in terms of layers—a prerequisite skill for deriving the formula ($V = l \cdot w \cdot h$) used for finding the volume of a prism.



Relate Perimeter, Area, and Volume The exercises in Lesson 12 help students compare and contrast perimeter, area, and volume.

Tell if you need to measure for length, area, or volume. Then write the number of measurements you need to make.

1. How tall are you? length; 1
2. How much carpet is needed for a floor? area; 2
3. How much sand is in a sandbox? volume; 3
4. How much wallpaper is needed for one wall? area; 2
5. How long is a string? length; 1
6. How much space is there inside a refrigerator? volume; 3



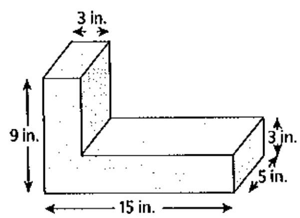
Lesson 12 also gives students an opportunity to apply their understanding of perimeter, area, and volume in real world contexts.

9. Soledad has a storage box. The box is $6\frac{1}{2}$ inches long, $4\frac{3}{4}$ inches wide, and 7 inches tall. She wants to run a border around the top of the box. How much border does she need?
   $22\frac{1}{2}$ in.

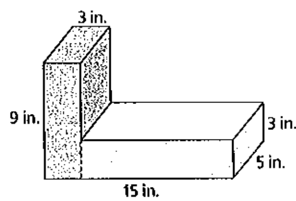
10. The refrigerator is $5\frac{2}{3}$ feet tall, $2\frac{2}{7}$ feet wide, and $2\frac{1}{4}$ feet deep. How much space does the refrigerator take up on the floor?
    $5\frac{1}{7}$ sq ft

11. Melissa is stacking storage cubes in a crate. The bottom of the crate is 8 inches by 12 inches. The volume of the crate is 768 cubic inches. If a storage cube has a length of 4 inches, how many storage cubes will fit in a crate?
    12 storage cubes

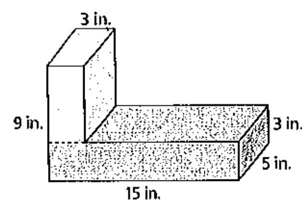
Composite Solid Figures In Lesson 13, students extend their understanding of finding volume to finding the volume of composite solids made of two or more rectangular prisms. This involves decomposing the composite figures. Initially, colors are used to help students see the individual sections that are part of the composite solid. Example 1 and Example 2 illustrate two possible ways to decompose the solid. After finding the volume, students verify that the volume is the same no matter how they break the figure apart.



Example 1



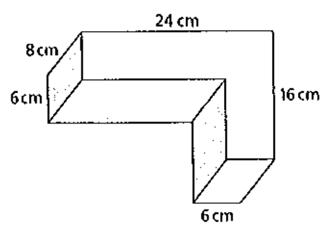
Example 2



Students progress to finding the volume of composite figures that do not use shading to aid decomposition, and they solve real world problems.

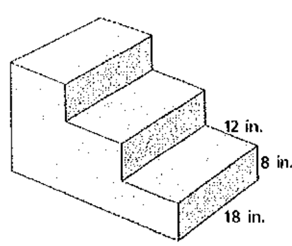
Find the volume of each composite figure.

4.



$V =$ 1,632 cubic centimeters

9. An in-ground swimming pool often has steps that are made from poured concrete. In the sketch of the steps at the right, the steps are identical, each measuring 18 inches from side to side, 12 inches from front to back, and 8 inches tall.



Calculate the amount of concrete that is needed to form the steps.

10,368 cubic inches

Two-Dimensional Figures

Lessons 14 15 16

Attributes of Quadrilaterals Lesson 14 provides opportunities to build reasoning and classifying skills with respect to quadrilaterals.

Write *true* or *false*. If the statement is false, sketch a counterexample.
Sketches will vary. Samples are given.

1. All quadrilaterals have at least one pair of parallel sides.

   false

2. All squares have adjacent sides that are perpendicular.

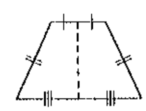
   true

Sketch a shape that fits the description if possible.
Sketches will vary. Samples are given.

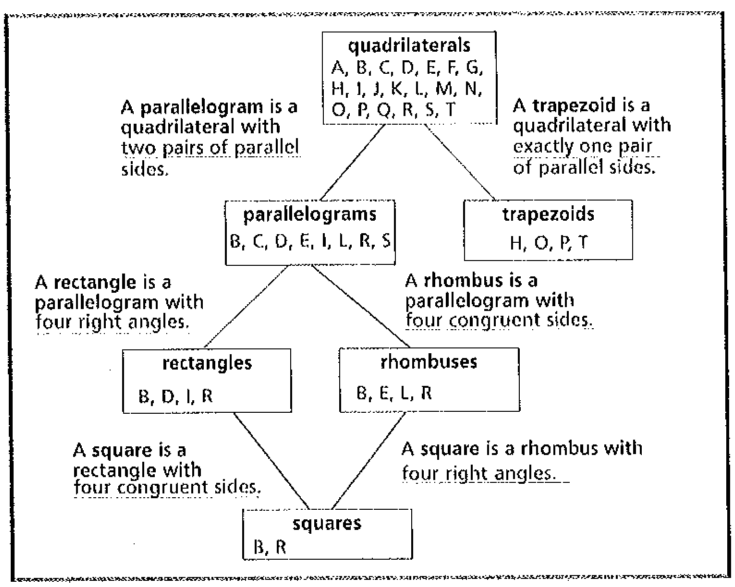
7. a parallelogram with exactly two right angles

   not possible

8. a trapezoid with one line of symmetry



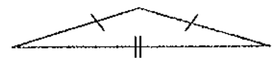
Students also study how the categories of quadrilaterals are related, sorting shapes into each category (or multiple categories) and then completing statements about the categories.



Attributes of Triangles The activities in Lesson 15 extend reasoning and classifying skills to include triangles.

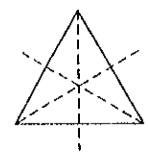
3. All right triangles have two acute angles.
   true

4. Any triangle with an obtuse angle must be scalene.
   false

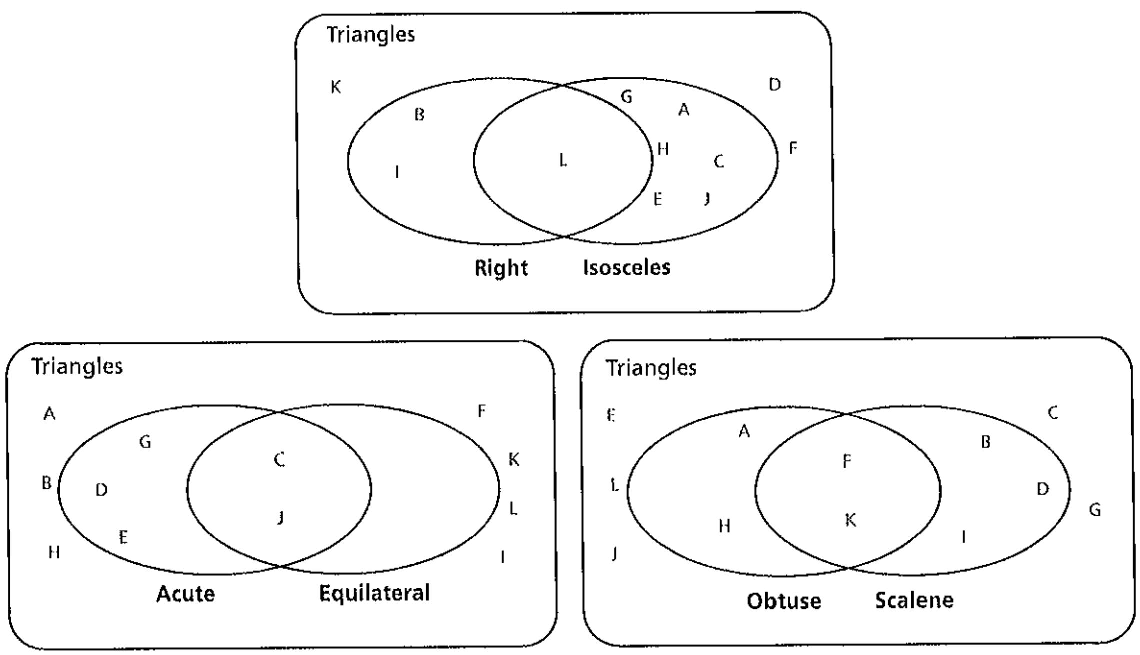


8. a triangle with two right angles
   not possible

9. a triangle with more than one line of symmetry



Students also study how the categories of triangles are related, sorting shapes in diagrams.



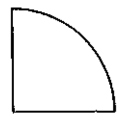
Attributes of Other Two-Dimensional Shapes The activities in Lesson 16 extend reasoning and classifying skills to include a variety of polygons.

1. 

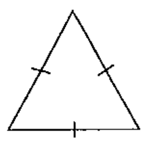
not a polygon; not closed

3. 

polygon

5. 

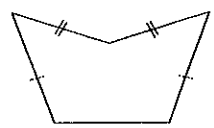
not a polygon; not made from segments

7. 

triangle

regular   not regular

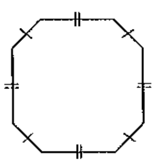
concave   convex

8. 

pentagon

regular   not regular

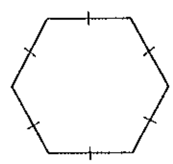
concave   convex

9. 

octagon

regular   not regular

concave   convex

10. 

hexagon

regular   not regular

concave   convex

Focus on Mathematical Practices

Lesson 17

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about perimeter, area, and volume to solve problems related to aquariums.